Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS332: Midterm Examination (Set I)

COURSE : ECS332 (Principles of Communications)
DATE : August 1, 2011
SEMESTER : 1/2011
INSTRUCTOR: Dr.Prapun Suksompong
TIME : 13:30-16:30
PLACE : BKD 3507

Name		ID	
		Seat	

Instructions:

1. Including this cover page, this exam has 12 pages.
2. **Read these instructions and the questions carefully**.
3. Fill out the form above.
4. **Closed book. Closed notes**.
5. **Basic calculators**, e.g. FX-991MS, **are permitted**, but borrowing is not allowed.
6. Unless specified otherwise, ***write down all the steps*** that you have done to obtain your answers. You may not get any credit even when your final answer is correct without showing how you get your answer.
7. Allocate your time wisely.
8. Do not forget to write your **first name and the last three digits of your ID** in the spaces provided on the top of each examination page, starting from page 2.
9. Allocate your time wisely.
10. When not explicitly stated/defined, all notations and definitions follow ones given in lecture.
11. Some points are reserved for reducing answers into their simplest forms.
12. Do not cheat. The use of communication devices including mobile phones is prohibited in the examination room.
13. Do not panic.

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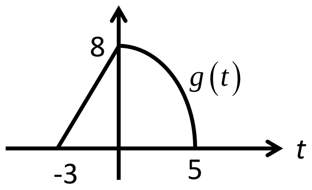
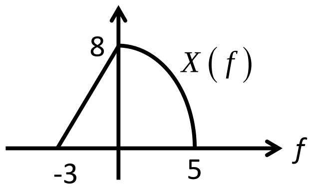
1. (5 pt) For the signal $g(t)$ shown below. Sketch $g(4-2t)$. No explanation is needed.



2. (5 pt) The Fourier transform $X(f)$ for a signal $x(t)$ is shown below. Let $y(t)=x(4-2t)$. Sketch $|Y(f)|$. No explanation is needed.



3. (5 pt) The Fourier transform of the Gaussian pulse $g(t)=e^{-\pi t^2}$ is given by $G(f)=e^{-\pi f^2}$. Using this information and the time-scaling property, find the Fourier transform of $x(t)=e^{-3t^2}$.

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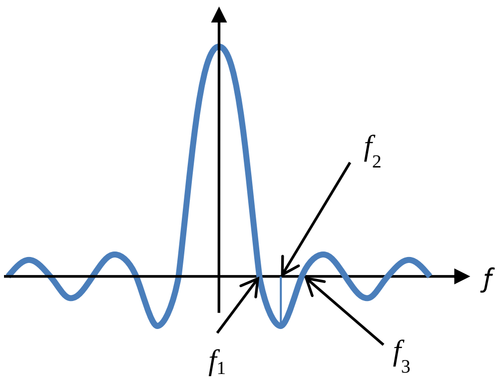
4. (5 pt) Suppose $g(t) = 1[|t| \le 2]$.
   a. (2 pt) Find $G(f)$.

   b. (2 pt) Find the smallest positive zero-crossing of $G(f)$.

   c. (1 pt) A non-accurate plot of $G(f)$ is given below. Your answer in part (b) is denoted by f_1. The next smallest positive zero-crossing of $G(f)$ is denoted by f_3. Between f_1 and f_3, there is a local minimum at f_2.

   Compare f_2 and $\frac{f_1 + f_3}{2}$. Theoretically, should they be the same? If not, which one is larger?



5. (5 pt) In this question, you are provided with a partial proof of an important result in the study of Fourier transform. Your task is to figure out the quantities/expressions inside the boxes labeled a,b,c, and d. Put your answers in the spaces provided at the end of the question. No explanation is needed.

   We start with a function $g(t)$. Then, we define $x(t) = \sum_{\ell=-\infty}^{\infty} g(t - \ell T)$. It is a sum that involves $g(t)$. What you will see next is our attempt to find another expression for $x(t)$ in terms of a sum that involves $G(f)$.

To do this, we first write $x(t)$ as $x(t)=g(t)*\sum_{\ell=-\infty}^{\infty}\delta(t-\ell T)$. Then, by the convolution-in-time property, we know that $X(f)$ is given by

$$X(f)=G(f)\times\boxed{\quad a \quad}\sum_{\ell=-\infty}^{\infty}\delta\left(f+\boxed{\qquad b \qquad}\right).$$

We can get $x(t)$ back from $X(f)$ by the inverse Fourier transform formula: $x(t)=\int_{-\infty}^{\infty}X(f)e^{j2\pi ft}df$. After plugging in the expression for $X(f)$ from above, we get

$$x(t)=\int_{-\infty}^{\infty}e^{j2\pi ft}G(f)\boxed{\quad a \quad}\sum_{\ell=-\infty}^{\infty}\delta\left(f+\boxed{\qquad b \qquad}\right)df$$

$$=\boxed{\quad a \quad}\int_{-\infty}^{\infty}\sum_{\ell=-\infty}^{\infty}e^{j2\pi ft}G(f)\delta\left(f+\boxed{\qquad b \qquad}\right)df.$$

By interchanging the order of summation and integration, we have

$$x(t)=\boxed{\quad a \quad}\sum_{\ell=-\infty}^{\infty}\int_{-\infty}^{\infty}e^{j2\pi ft}G(f)\delta\left(f+\boxed{\qquad b \qquad}\right)df.$$

We can now evaluate the integral via the sifting property of the delta function and get

$$x(t)=\boxed{\quad a \quad}\sum_{\ell=-\infty}^{\infty}e^{\boxed{\ c\ }}G\left(\boxed{\quad d \quad}\right).$$

a = ________________	c = ________________
b = ________________	d = ________________

6. (10 pt) You are given the baseband signal $m(t)=3\cos(500\pi t)+\cos(1000\pi t)$.
   a. (5 pt) Sketch the spectrum of $m(t)$. No explanation is needed.

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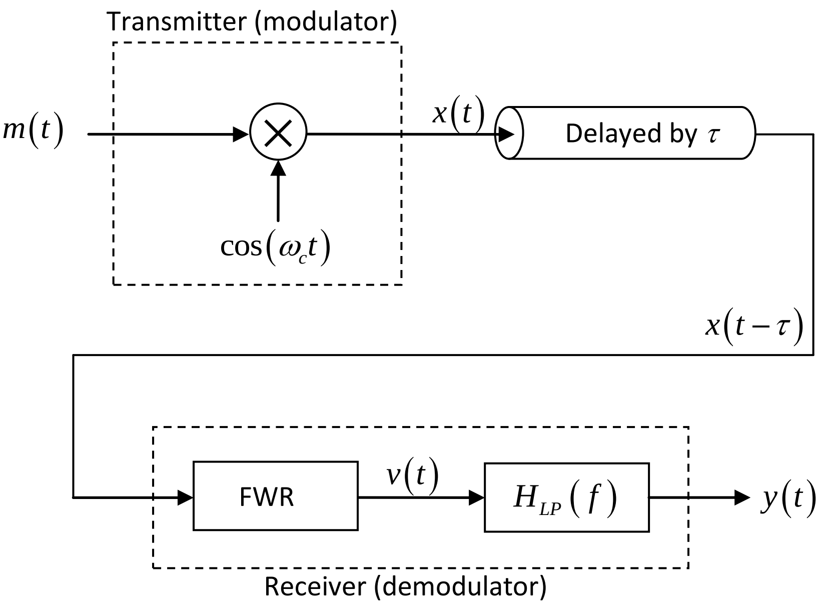
b. (5 pt) Sketch the spectrum of the DSB-SC signal $m(t)\cos(5{,}000\pi t)$. No explanation is needed.

7. (8 pt) Suppose $m(t) \xrightarrow{\mathcal{F}} M(f)$ is bandlimited to W, i.e., $|M(f)| = 0$ for $|f| > W$. Consider the following DSB-SC transceiver.



Also assume that $f_c \gg W$ and that $H_{LP}(f) = \begin{cases} 1, & |f| \le W \\ 0, & \text{otherwise.} \end{cases}$

Make an extra assumption that $m(t) \ge 0$ for all time t and that the **full**-wave rectifier (FWR) input-output relation is described by a function $f_{FWR}(\bullet)$:

$$f_{FWR}(x) = \begin{cases} x, & x \ge 0, \\ -x, & x < 0. \end{cases}$$

(Questions start on the next page.)

a. Recall that the ***half***-wave rectifier input-output relation is described by a function $f_{HWR}(\bullet)$: $f_{HWR}(x) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0. \end{cases}$

We have seen in class and in HW2 that when the receiver uses ***half***-wave rectifier,

$$v(t) = x(t-\tau) \times g_{HWR}(t-\tau),$$

where $g_{HWR}(t) = 1[\cos(\omega_c t) \geq 0]$.

i. (3 pt) The receiver in this question uses **full**-wave rectifier. Its $v(t)$ can be described in a similar manner; that is

$$v(t) = x(t-\tau) \times g_{FWR}(t-\tau).$$

Find $g_{FWR}(t)$.

Hint: $g_{FWR}(t) = c_1 \times g_{HWR}(t) + c_2$ for some constants c_1 and c_2. Can you find these constants?

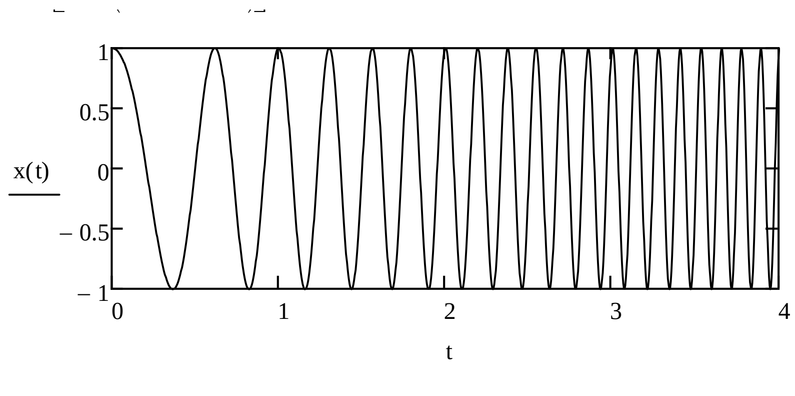
ii. (2 pt) Recall that the Fourier series expansion of $g_{HWR}(t)$ is given by

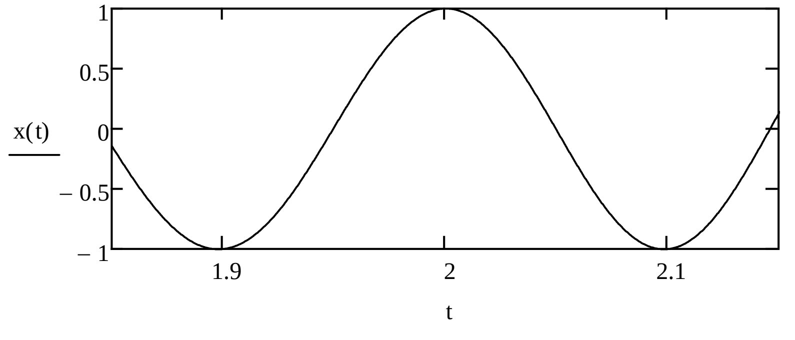
$$g_{HWR}(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \frac{1}{7}\cos 7\omega_c t + \ldots\right).$$

Find the Fourier series expansion of $g_{FWR}(t)$.

b. (3 pt) Find $y(t)$ (the output of the LPF).

8. (12 pt) The signal $x(t) = \cos\left(2\pi\left(t^2 + at + 3\right)\right)$ is plotted below. The time unit is in seconds.





a. (2 pt) Find the instantaneous frequency (in Hz) at $t = 2$. Hint: it is an integer.

b. (5 pt) Find the value of a. Hint: it is an integer.

c. (5 pt) Find the instantaneous frequency (in Hz) at $t = 5$. Hint: it is an integer.

9. (10 pt) A modulated signal with carrier frequency $f_c = 10^5$ Hz is described by the equation

$$x(t) = 10\cos\left(2\pi f_c t + 5\sin\left(3000t\right)\right).$$

a. (5 pt) Find the instantaneous frequency $f(t)$ of $x(t)$.

b. (5 pt) Estimate the bandwidth of $x(t)$ via Carson's rule.

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10. (8 pt) A modulated signal with carrier frequency $f_c = 10^5$ Hz is described by the equation

$$x(t) = 10\cos\left(2\pi f_c t + 5\sin(3000t) + 10\sin(2000\pi t)\right).$$

a. (5 pt) Find the instantaneous frequency $f(t)$ of $x(t)$.

b. (3 pt) Estimate the bandwidth of $x(t)$ via Carson's rule.

11. (16 pt) Determine the Nyquist sampling rate and for the signals in the table below. No explanation is needed.

	Nyquist sampling rate	Nyquist sampling interval
$\text{sinc}(200\pi t)$		
$\text{sinc}^2(200\pi t)$		
$\text{sinc}(200\pi t) + 5\,\text{sinc}^2(120\pi t)$		
$\text{sinc}(100\pi t)\,\text{sinc}(200\pi t)$		

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12. (5 pt) Let $c(t) = \begin{cases} A, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases}$

You may assume that $A > 0$.

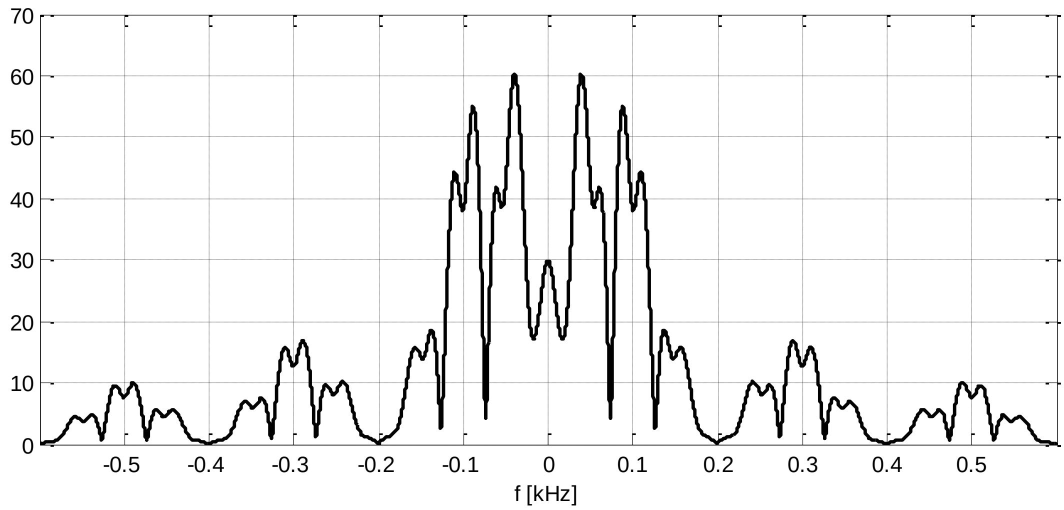
A message $m = (m_0, m_1, m_2, \ldots, m_9) =$

(-1 1 1 -1 -1 -1 1 -1 1 -1)

is sent via

$$s(t) = \sum_{k=0}^{\ell-1} m_k c(t - kT) \text{ where } \ell \text{ is the length of } m.$$

The magnitude $|S(f)|$ of the spectrum is plotted below.



a. (2 pt) Find T.

b. (3 pt) Find A.

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13. (6 pt) Use properties of Fourier transform to evaluate the following integrals. (Recall that $\operatorname{sinc}(x) = \frac{\sin x}{x}$.) Clearly state which properties that you use.

a. (3 pt) $\int_{-\infty}^{\infty} \operatorname{sinc}\left(\sqrt{5}x\right) dx$

b. (1 pt) $\int_{-\infty}^{\infty} e^{-2\pi f \times 2j} 2\operatorname{sinc}(2\pi f)\left(e^{-2\pi f \times 5j} 2\operatorname{sinc}(2\pi f)\right)^{*} df$

c. (1 pt) $\int_{-\infty}^{\infty} \operatorname{sinc}\left(\sqrt{5}x\right)\operatorname{sinc}\left(\sqrt{7}x\right) dx$

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d. (1 pt) $\int_{-\infty}^{\infty} \operatorname{sinc}(\pi(x-5))\operatorname{sinc}\left(\pi\left(x-\frac{7}{2}\right)\right)dx$

$$2\cos^2 x = 1+\cos(2x)$$
$$2\sin^2 x = 1-\cos(2x)$$
$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}dt$$
$$\cos(2\pi f_c t+\theta) \xrightleftharpoons{\mathcal{F}} \frac{1}{2}\delta(f-f_c)e^{j\theta} + \frac{1}{2}\delta(f+f_c)e^{-j\theta}$$
$$g(t-t_0) \xrightleftharpoons{\mathcal{F}} e^{-j2\pi ft_0}G(f)$$
$$e^{j2\pi f_0 t}g(t) \xrightleftharpoons{\mathcal{F}} G(f-f_0)$$
$$g(t)\cos(2\pi f_c t) \xrightleftharpoons{\mathcal{F}} \frac{1}{2}G(f-f_c)+\frac{1}{2}G(f+f_c)$$
$$1\left[|t| \le a\right] \xrightleftharpoons{\mathcal{F}} 2a\operatorname{sinc}(2\pi fa) \text{ where } \operatorname{sinc}(x) = \frac{\sin x}{x}$$

Extra Credits:

Complete the following lyrics of the “Fourier's Song”

Integrate your function times a complex ____________
It's really not so hard you can do it with your pencil
And when you're done with this calculation
You've got a brand new function - the Fourier Transformation
What a prism does to sunlight, what the ear does to sound
Fourier does to signals, it's the coolest trick around
Now filtering is easy, you don't need to _____________
All you do is multiply in order to solve.

From time into frequency - from frequency to time

Every operation in the time domain
Has a Fourier analog - that's what I claim
Think of a delay, a simple _____________ in time
It becomes a phase rotation - now that's truly sublime!
And to differentiate, here's a simple trick
Just multiply by j omega, ain't that slick?
Integration is the inverse, what you gonna do?
Divide instead of multiply - you can do it too.